PROBLEMS :

1. Find the inverse of the given matrix by Gauss - Jordon method $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

Solution :-

$$[A, I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & -3 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -4 & -1 & -1 & 1 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to R_3 - R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 4 & 0 & -5 & 3 & 1 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & \frac{-1}{4} \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_2 \\ R_2 \to 4R_2 + R_3 \\ R_3 \to R_3 / -4 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{7}{4} & -\frac{5}{4} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{5}{4} & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2/4 \end{array}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 7/4 & -5/4 & 1/4 \\ -5/4 & 3/4 & 1/4 \\ 1/4 & 1/4 & -1/4 \end{bmatrix}$$

2) Using Gauss-Jordan method, find the inverse of

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

Solution:-

$$\text{Let } [A, I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right] \quad R_2 \to \frac{R_2}{2}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 3/2 & -1/2 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_2 \\ \\ R_3 \to R_3 + 2R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 3/2 & -1/2 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right] \quad R_3 \to \frac{R_3}{-4}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right] \begin{array}{l} R_1 \to R_1 - 6R_3 \\ R_2 \to R_2 + 3R_3 \\ \end{array}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 12 & 4 & 6 \\ -5 & -1 & -3 \\ -1 & -1 & -1 \end{bmatrix}$$

Verification :

$$AA^{-1} = I$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3). Using Gauss-Jordan method, find the inverse of the matrix.
$$\begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$$

Solution :

$$\text{Let } [A, I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 6 & 1 & 0 & 0 \\ 2 & 5 & 15 & 0 & 1 & 0 \\ 6 & 15 & 46 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 6 & 1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 3 & 10 & -6 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 6R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 6 & 1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 1 \end{array}\right] R_3 \to R_3 - 3R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 18 & -6 \\ 0 & 1 & 0 & -2 & 10 & -3 \\ 0 & 0 & 1 & 0 & -3 & 1 \end{array}\right] \begin{array}{l} R_1 \to R_1 - 6R_3 \\ R_2 \to R_2 - 3R_3 \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -2 & 0 \\ 0 & 1 & 0 & -2 & 10 & -3 \\ 0 & 0 & 1 & 0 & -3 & 1 \end{array}\right] \quad R_1 \to R_1 - 2R_2$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 10 & -3 \\ 0 & -3 & 1 \end{bmatrix}$$

Homework problems :-

① Find the inverse of $A = \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$ by Gauss-Jordan method.

Ans: $A^{-1} = \frac{1}{8}\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$.

②. Find the inverse of $A = \begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}$ using Gauss - Jordan method.

Ans:

$$A^{-1} = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$$